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Quantum Cooling Evaporation Process in Regular Black Holes

Yun Soo Myung[a,1], Yong-Wan Kim [b,2], and Young-Jai Park[c,3]

[a]*Institute of Mathematical Science and School of Computer Aided Science, Inje University, Gimhae 621-749, Korea*

[b] *National Creative Research Initiative Center for Controlling Optical Chaos, Pai-Chai University, Daejeon 302-735, Korea*

[c]*Department of Physics and Center for Quantum Spacetime, Sogang University, Seoul 121-742, Korea*

Abstract

We investigate a universal behavior of thermodynamics and evaporation process for the regular black holes. We newly observe an important point where the temperature is maximum, the heat capacity is changed from negative infinity to positive infinity, and the free energy is minimum. Furthermore, this point separates the evaporation process into the early stage with negative heat capacity and the late stage with positive heat capacity. The latter represents the quantum cooling evaporation process. As a result, the whole evaporation process could be regarded as the inverse Hawking-Page phase transition.

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[1]e-mail address: ysmyung@inje.ac.kr

[2]e-mail address: ywkim@pcu.ac.kr

[3]e-mail address: yjpark@sogang.ac.kr

Introduction.—Hawking's semiclassical analysis of the black hole radiation suggests that most information about initial states is shielded behind the event horizon and will not back to the asymptotic region far from the evaporating black hole [1]. This means that the unitarity is violated by an evaporating black hole. However, this conclusion has been debated by many authors for three decades [2, 3, 4]. It is closely related to a long standing puzzle of the information loss paradox, which states the question of whether the formation and subsequent evaporation of a black hole is unitary. One of the most urgent problems in black hole physics is to resolve the unitarity issue. In this direction a complete description of black hole evaporation is an important issue. In order to determine the final state of evaporation process, a more precise treatment including quantum gravity effects and backreaction is generally required. At present, two leading candidates for quantum gravity are the string theory and the loop quantum gravity. Interestingly, the semiclassical analysis of the loop quantum black hole provides a regular black hole (RBH) without singularity in contrast to the classical one [5]. Its minimum size r_c is at Planck scale ℓ_{Pl}. On the other hand, in the continuing search for quantum gravity, the black hole thermodynamics may be related to a future experimental result at the LHC [6].

RBHs have been considered, dating back to Bardeen [7], for avoiding the curvature singularity beyond the event horizon in black hole physics [8]. Their causal structures are similar to the Reissner-Nordström (RN) black hole with the singularity replaced by de Sitter space-time with curvature radius $r_0 = \sqrt{3/\Lambda}$ [9]. Recently, Hayward has discussed the formation and evaporation process of a RBH with minimum size l [10], which can be identified with the minimal length induced from the string theory [11]. A more rigorous treatment of the evaporation process was carried out for the renormalization group (RG) improved black hole with minimum size $r_{cr} = \sqrt{\tilde{\omega} G}$ [12]. The noncommutativity also provides another RBH with minimum scale $\sqrt{\theta}$: noncommutative black hole [13]. Very recently, we have investigated thermodynamics and evaporation process of the noncommutative black hole [14]. The RN black hole with charge Q also belongs to the RBH [15], even though it has a timelike singularity [16]. It turned out that the final state of the evaporation process for all RBHs is a cold, Planck size remnant of the extremal black hole. The connection between their minimum sizes is given by $r_c \sim r_0 \sim l \sim r_{cr} \sim \sqrt{\theta} \sim Q \sim \ell_{Pl}$.

It is very important to study the terminal phase of black hole evaporation. In the semiclassical study of the Schwarzschild black hole, the temperature ($T_H \propto 1/m$) and the luminosity ($L_{Sch} \propto 1/m^2$) diverge as m approaches zero. This means that the semiclassical approach breaks down for very light holes. Furthermore, one has to take into account the backreaction. It was shown that the effect of quantum gravity could cure this pathological short distance behavior [17].

In this Letter, we first study universal thermodynamic properties of RBHs by analyzing the minimal model proposed by Hayward [10] and then investigate its evaporation process. We wish to remind the reader that the RBH is closely related to effects of quantum gravity. Especially, we newly observe an important point at $r_+ = r_m$ where the temperature is maximum, the heat capacity is changed from negative infinity to positive infinity, and the free energy is minimum. This point separates the whole evaporation process into the early stage with negative heat capacity and the late stage with positive heat capacity. The latter is described by the quantum cooling evaporation process (QCEP) which is a necessary step to reach extremal black hole. For the QCEP, the temperature decreases near Planck scale as the mass of black hole decreases, while for the early evaporation process, the temperature increases as the mass of black hole decreases. It is important to note that we do not need to take into account the backreaction for RBHs due to the QCEP.

We could understand the thermodynamic process for RBHs from the analogy of the Hawking-Page (HP) phase transition in the AdS black hole [18, 19]. In the HP transition, we start with thermal radiation in AdS space. A small black hole appears with negative heat capacity. The heat capacity changes from negative infinity to positive infinity at the minimum temperature. Finally, the large black hole with positive heat capacity comes out as a stable object. There is a change of the dominance at the critical temperature: from thermal radiation to black hole.

On the other hand, we start with the large unstable black hole with negative heat capacity for RBHs. The heat capacity changes from negative infinity to positive infinity at the maximum temperature. Then, the small black hole with positive heat capacity comes out. There is a change of the dominance at the critical temperature near $T = 0$: from a large black hole to a different, extremal black hole. Consequently, we regard the thermodynamic process of RBHs as the inverse HP transition because this is the process from initial (unstable) large black hole to final (stable) extremal black hole. We note that the QCEP plays a crucial role in the inverse HP transition. However, it takes an infinite time to reach the final remnant of extremal black hole using the quantum-corrected Vaidya metric.

Thermodynamics of regular black holes.—It was shown that in order to obtain a RBH, we need to introduce an anisotropic fluid whose energy-momentum tensor is given by $T^{\mu}{}_{\nu} = \mathrm{diag}[-\rho, p_r, p_\perp, p_\perp]$ with energy density ρ, radial pressure p_r, and tangential pressure $p_\perp$. For simplicity, we study the minimal model provided by the energy-momentum tensor

$$\rho = \frac{3l^2m^2}{2\pi(r^3+2l^2m)^2} = -p_r, \quad p_\perp = \frac{3l^2m^2(r^3-l^2m)}{\pi(r^3+2l^2m)^3} \tag{1}$$

with the Planck units of $c=\hbar=G=\ell_{Pl}=1$. Solving the Einstein equation $G_{\mu\nu}=8\pi T_{\mu\nu}$ leads to the solution

$$ds^2_{RBH} \equiv g_{\mu\nu}dx^\mu dx^\nu = -F(r)dt^2 + F(r)^{-1}dr^2 + r^2 d\Omega_2^2. \tag{2}$$

The metric function $F(r)$ is given by

$$F(r) = 1 - \frac{2mr^2}{r^3+2l^2m}, \tag{3}$$

where l denotes the curvature radius of de Sitter space-time near the center and $m = 4\pi\int_0^\infty \rho(r)r^2dr$ represents the Arnowitt-Deser-Misner mass. We have de Sitter space-time $F(r) \simeq 1 - r^2/l^2$ as $r \to 0$, while an asymptotically Schwarzschild space-time $F(r) \simeq 1-2m/r$ appears as $r \to \infty$. Hence, ρ connects the de Sitter vacuum in the origin with the Minkowski vacuum at infinity.

From $F=0(m_\pm = r_\pm^3/2(r_\pm^2-l^2))$, we obtain the minimum mass $m_* = 3\sqrt{3}l/4$ and the minimum radius $r_* = \sqrt{3}l$. For definiteness, we consider three different types: i) For $m > m_*$, two distinct horizons appear with the inner cosmological horizon $r = r_-(l < r_- \leq r_*)$ and the outer event horizon $r = r_+(r_* \leq r_+ < \infty)$. They are analytically given by $r_+ = \frac{m}{3}(2+4\cos\frac{\alpha}{3})$, $r_- = \frac{m}{3}(2+4\cos(\frac{\alpha}{3}-\frac{2\pi}{3}))$ where $\cos\alpha = 1-\frac{2m_*^2}{m^2}$ with $\frac{2m_*}{m} < \alpha \leq \pi$. In particular, for $m \gg m_*(\alpha \to \frac{2m_*}{m} \simeq 0)$, the outer horizon is located at $r_+ \simeq 2m$, while the inner horizon is at $r_- \simeq l$. ii) In case of $m = m_*(\alpha=\pi)$, one has a degenerate horizon at $r = r_*$, which corresponds to the extremal black hole. iii) For $m < m_*$, there is no horizon.

The black hole temperature can be calculated to be

$$T_{RBH}(r_+) = \frac{1}{4\pi}\left[\frac{dF}{dr}\right]_{r=r_+} = \frac{1}{4\pi r_+}\frac{r_+^2-r_*^2}{r_+^2} \tag{4}$$

with a fixed core radius $l=1$. For $r_+^2 \gg 1$, one recovers the Hawking temperature $T_H = 1/4\pi r_+$ of the Schwarzschild black hole. Therefore, at the early stage of the Hawking radiation, the black hole temperature increases as the horizon radius decreases. It is important to investigate what happens as $r_+ \to 0$. In the Schwarzschild case, T_H diverges and this puts the limit on the validity of the evaporation process via the Hawking radiation. Against this scenario, the temperature T_{RBH} includes quantum effects, which are relevant at short distance comparable to the Planck scale of $r_+ \simeq 1$ [13]. As is shown in Fig.1, the temperature of the RBH grows during its evaporation until it reaches to the

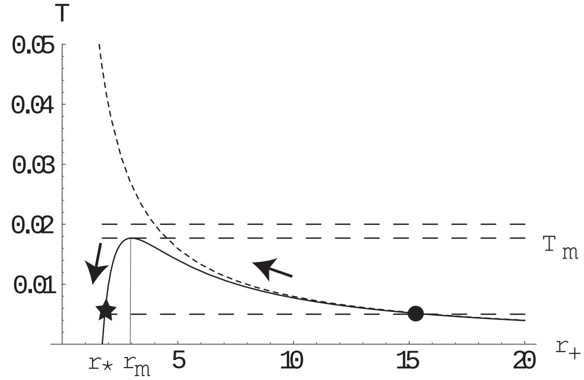

Figure 1: The solid line: temperature T_{RBH} with the maximum point at $r_+ = r_m$. The dashed curve is the Schwarzschild case of $1/4\pi r_+$. The QCEP takes place for $r_* \le r_+ < r_m$. Three horizontal dashed lines denote the temperature $T = 0.02, T_m$ and 0.005 from the top to the bottom. $\bullet(r_+ = r_i = 15.72)$ and $\star(r_+ = r_e = 1.84)$ represent the initial point and the end point for an evaporation process ($\rightarrow$), respectively.

maximum value $T_m = 0.017$ at $r_+ = r_m = 3(m = m_m = 1.68)$ and then falls down to zero at $r_+ = r_* = \sqrt{3}(m = m_*)$ which the extremal black hole appears with $T_* = 0$. As a result, the thermodynamics process is split into the right branch of $r_m \le r_+ < \infty$ called the early stage of evaporation and the left branch of $r_* \le r_+ < r_m$ called the QCEP. In the region of $r < r_*$, there is no black hole for $m < m_*$ and thus the temperature cannot be defined. For $m > m_*$, we have the inner horizon at $r = r_-$ inside the outer horizon but an observer at infinity does not recognize the presence of this cosmological horizon. Hence, we regard this region as the forbidden region. The entropy $S_{RBH} = \int_{r_*}^{r_+}(m'/T_{RBH})dr$ of the RBH can be obtained from the first-law of thermodynamics $dm = T_{RBH}dS_{RBH}$ as

$$S_{RBH}(r_+) = \frac{A}{4} + \frac{\pi}{50}\left[-\frac{10}{2r_+^2-1} + 108\ln(r_+^2 - r_*^2) + 17\ln(2r_+^2-1)\right] \tag{5}$$

with the area of the event horizon $A = 4\pi r_+^2$. We have negative infinity-entropy for the extremal black hole at $r_+ = r_*$ due to the third term. Hence we cannot find logarithmic correction to the extremal black hole. On the other hand, we have the area-law behavior of $S_{BH} \simeq \pi r_+^2$ for $r_+ \gg 1$.

In order to check the thermal stability of the RBH, we have to know the heat capacity. Its heat capacity $C_{RBH} = dm/dT_{RBH}$ is given by

$$C_{RBH}(r_+) = -2\pi r_+^2 \, \frac{r_+^4(r_+^2 - r_*^2)}{(r_+^2-1)^2(r_+^2 - r_m^2)} \tag{6}$$

and its variation is plotted in Fig. 2. Here we find a stable region of $C_{RBH} > 0$ which

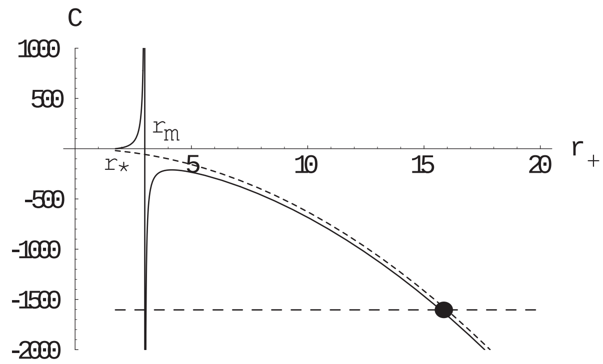

Figure 2: The solid line: heat capacity C_{RBH} as a function of the black hole radius r_+. The QCEP takes place for $r_* \leq r_+ < r_m (C \geq 0)$. The dashed curve is the Schwarzschild case of $-2\pi r_+^2$. The horizontal dashed line denotes the initial heat capacity $C_i = -1604(\bullet)$ for an evaporation process.

represents the QCEP. This means that the RBH could be thermodynamically stable in the range of $r_* < r_+ < r_m$. The heat capacity becomes singular at $r_+ = r_m$ which corresponds to the maximum temperature $T = T_m$. We also observe that a thermodynamically unstable region ($C_{RBH} < 0$) appears for $r_+ > r_m$. We note that in the Hawking regime of $r_+ \gg 1$, C_{RBH} is consistent with the specific heat $C_{RBH} \simeq -2\pi r_+^2$ of the Schwarzschild black hole. Also we have $C_{RBH}|_{r_+=r_*} = 0$ for the extremal black hole.

Now, we are in a position to discuss a possible phase transition. For this purpose, we introduce the on-shell free energy as

$$F^{on}_{RBH}(r_+) = m(r_+) - T_{RBH}(r_+)S_{RBH}(r_+). \tag{7}$$

Its graph is shown in Fig. 3. Interestingly, the free energy has the minimum value at $r_+ = r_m$. The QCEP takes place for $r_* \leq r_+ < r_m$. For $r_+ \gg 1$, one recovers $F^{on}_{RBH} \simeq r_+/4$ for the Schwarzschild black hole. Further, one needs to know the off-shell free-energy

$$F^{off}_{RBH}(r_+, T) = m(r_+) - TS_{RBH}(r_+) \tag{8}$$

with the temperature T of the heat reservoir. Finally, let us describe the inverse HP phase transition, which is closely related to the evaporation process of the RBH. For $T = 0.02 > T_m$, there is no junction between F^{on}_{RBH} and F^{off}_{RBH} except $r_+ = r_*$. For $T = T_m$, we find one junction point (the maximum point) at $r_+ = r_m$. For $T = 0.005 < T_m$, we find two junction points: large black hole at $r_+ = r_i$ and extremal black hole at $r_+ = r_*$. Actually, there is a change of dominance at the critical temperature $T = 0.005$: from unstable large black hole to stable extremal back hole. The off-shell (non-equilibrium)

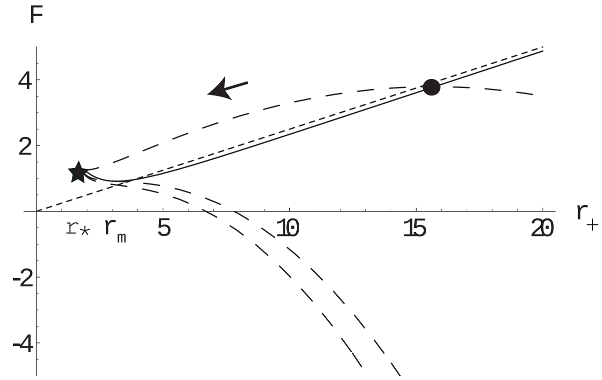

Figure 3: The solid line: plot of the free energy F^{on}_{RBH} as a function of r_+. The small-dashed line is the Schwarzschild case of $r_+/4$. The large-dashed curves denote $F^{off}_{RBH}(r_+, T = 0.005)$, $F^{off}_{RBH}(r_+, T = T_m)$, $F^{off}_{RBH}(r_+, T = 0.02)$ from the top to the bottom. $\bullet(r_+ = r_i = 15.72)$ and $\star(r_+ = r_e = 1.84)$ represent the initial point and the end point for an evaporation process ($\rightarrow$), respectively.

process starts with $r_+ = r_i = 15.72$ and ends at $r_+ = r_e = 1.84$. We observe that $r_i \to \infty$ and $r_e \to r_*$, as $T \to 0$. Hence this could be regarded as the inverse HP transition for the RBH.

Evaporation of the regular black holes.— We start with the fact that the RBH looks like the RN black hole with the singularity replaced by a regular center. The evaporating process will terminate at the point which corresponds to the maximum cosmological horizon and the minimum event horizon ($r_- = r_+ = r_*$). Hence, it is interesting to explore the evaporation process of the RBH. We suggest that the dynamic regions are Vaidya-like with the negative-energy flux during the evaporation.

We begin by reexpressing the metric in Eq.(2) in terms of ingoing Eddington-Finkelstein coordinates (v, r, θ, ϕ). We introduce the advanced time coordinate: $v = t + \tilde{r}$ with $\tilde{r} = \int^r dr'/F(r')$. Using $dv = dt + dr/F(r)$, we obtain $ds^2 = -F(r)dv^2 + 2dvdr + r^2 d\Omega_2^2$. Considering the static metric together with the Stefan's law, the mass dependence of the luminosity is given by

$$L(m) = \sigma A T^4_{RBH} \tag{9}$$

with $\sigma = \pi^2/60$ for a single massless field with 2 degrees of freedom [12]. Now, we compute the mass $m(v)$ of the black hole as seen by a distant observer at time v. We solve the differential equation,

$$-m'(v) = L(m(v)), \tag{10}$$

where the prime denotes the differentiation with respect to v. The quantum-corrected Vaidya (QCV) metric is obtained by replacing the constant m in $F(r)$ with $m(v)$: $ds^2_{QCV} = -F(r,v)dv^2 + 2dvdr + r^2 d\Omega^2$ with

$$F(r,v) = 1 - \frac{2m(v)r^2}{r^3 + 2m(v)}. \tag{11}$$

For $m(v) \gg m_m$, Eq.(11) becomes the Vaidya metric, which was frequently used to explore the influence of the Hawking radiation on the Schwarzschild geometry [7, 20]. It is a solution to the Einstein equation $G_{\mu\nu} = 8\pi \tilde{T}_{\mu\nu}$, where $\tilde{T}_{\mu\nu}$ describes an inward moving null fluid. In this picture, the decreasing mass $m(v)$ is due to the inflow of negative energy.

It is instructive to ask which energy-momentum tensor $\tilde{T}_{\mu\nu}$ would give rise to the QCV metric. Computing the Einstein tensor with Eq.(11), a new component is given by

$$\tilde{T}^{v}{}_{r} = \frac{2r^4 m'(v)}{(r^3 + 2m(v))^2} \tag{12}$$

which describes the inflow of negative energy into the black hole for decreasing mass $m' < 0$. This shows pure radiation, recovering the Vaidya solution for the early stage of the evaporation. In the Vaidya case, the ingoing radiation creates a singularity. However, the center remains regular with de Sitter space-time for the RBHs. This implies that the quantum effects protect the core.

Both the early and the late stages of the evaporation process can be described by $F(r,v)$. We assume that a black hole starts with $m(v=0) = m_i > m_m$ in the Hawking regime. At the early stage, we use the known result of the Schwarzschild black hole: $T_H(m) = 1/8\pi m$, $L_{Sch}(m) = b/m^2$ with $b = \frac{\sigma}{256\pi^3}$. It is easy to solve the differential equation (10) for $L_{Sch}(m)$. The solution takes the form

$$m_{Sch}(v) = \Big[m_i^3 - 3bv\Big]^{1/3}. \tag{13}$$

This decreasing mass is valid during the early stage of the evaporation process, as long as $m(v) \geq m_m$. If one extrapolates Eq.(13) to small mass of $m(v) < m_m$, one finds $m(v_{Sch}) = 0$. This implies that a final explosion with $T \to \infty$ and $L \to \infty$ occurs, after a finite time of $v_{Sch} = m_i^3/(3b)$. However, this is not the case for the RBH.

The late stage of the evaporation process for the RBH is totally different from the Schwarzschild case. Instead, this is described by the QCEP. Rewriting Eq.(4) in terms of m near $m = m_*$ and together with Eq.(9), we obtain the approximate forms:

$$T_{RBH}(m) \simeq \alpha(m - m_*), \tag{14}$$

$$L_{RBH}(m) \simeq \beta(m - m_*)^4 \tag{15}$$

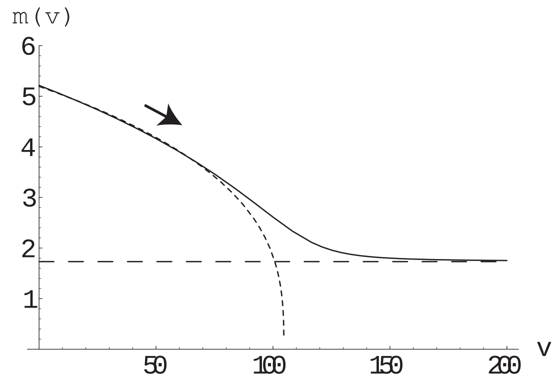

Figure 4: The solid line: plot of the decreasing mass $m(v)$ as a function of v. The small-dashed curve is for the Schwarzschild case of $m_{Sch}(v)$. The large-dashed line denotes $m_* = \sqrt{3}$.

with $\alpha = 3/8\pi m_*^2 = 0.07$ and $\beta = \sigma A \alpha^4 = 9\sigma/64\pi^3 m_*^6 = 0.00016$. Solving Eq. (10) with Eq. (15), one finds

$$m(v) - m_* \simeq \frac{m_1 - m_*}{[1 + 3\beta(m_1 - m_*)^3(v - v_1)]^{1/3}} \tag{16}$$

where v_1 is a time in the quantum region and $m_1 = m(v_1)$. For $v \to \infty$, the difference of $m(v) - m_*$ vanishes as $v^{-1/3}$. It was shown that $m(v) - m_*$ vanishes as v^{-1} for the RG-improved Vaidya metric [12] and $v^{-1/3}$ for the noncommutative-improved Vaidya metric. Hence, we obtain the late stage of the evaporation process: $T_{\rm RBH}(v) \propto v^{-1/3}$ and $L_{RBH} \propto v^{-4/3}$. We confirm that the RBHs lead to concrete predictions on the final state of the evaporation process. We note again that $m = m_*$ is the mass of a cold remnant, which is an extremal black hole with the Planck size. We solve Eq.(10) numerically and its plot is shown in Fig. 4. It shows an infinite time to reach the extremal black hole, in compared with the Schwarzschild black hole. The latter takes a finite time for a complete evaporation.

Discussions.—There are a lot of approaches for treating black hole evaporation process. This process is related to quantum gravitational effect, and its understanding provides a suitable framework toward a complete formulation of quantum gravity. In this work, we have focused on the thermodynamic approach to the late stage of the RBH evaporation. Our results and corresponding figures indicate stable features when the mass of the RBH becomes the Planck scale. We newly observe an important point where the temperature is maximum, the heat capacity is changed from negative infinity to positive infinity, and the free energy is minimum. We have obtained a maximum temperature $T = T_m$ that the RBH can reach before cooling down to absolute zero ($T = 0$). This point separates the whole evaporation process into the early stage with negative heat capacity and the late stage with positive heat capacity. The latter represents the QCEP, which is characterized by decreasing temperature, decreasing (positive) heat capacity, and increasing (on-shell) free energy. We note that the late stage of the evaporation process takes an infinite time to reach the final remnant of the extremal black hole using the quantum-corrected Vaidya metric.

Concerning the temperature T which defines the inverse Hawking-Page transition, we have still some arguments for regarding T as the temperature of heat reservoir. This is because we did not introduce any reservoir such as the cavity for the Schwarzschild black hole [21] and the negative cosmological constant for the AdS black hole [22]. These are necessary devices to derive the Hawking-Page transition from thermal radiation to large stable black hole. Here we have used the external temperature T without referring the reservoir. If one introduces the heat reservoir with temperature T, our evaporation process holds for $T_i < T = T_{RBH}$. On the other hand, for $T_i > T = T_{RBH}$, it evolves to a large stable black hole by absorbing the radiation in the heat reservoir continuously. Consequently, the whole evaporation process for RBHs could be regarded as the inverse Hawking-Page phase transition.

In conclusion, our result is universal for any RBHs, although we have investigated the thermodynamics and the evaporation process by choosing a minimal model suggested by Hayward. This is because the temperature in Fig. 1, the heat capacity in Fig. 2, and the free energy in Fig. 3 show the same behaviors for all known RBHs including the loop quantum and RN black holes. In this work the backreaction effect is trivial because the temperature approaches zero (not divergent) as $m \to m_*$. For the Schwarzschild case, one expects relevant backreaction effects during the terminal stage of the evaporation because of huge increase of temperature as is shown in Fig. 1. However, there is a suppression of quantum backreaction for the RBH, since it emits less and less energy as the QCEP does.

Acknowledgement

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